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THE $B \rightarrow K^*\psi$ POLARIZATION PUZZLE

R. Aleksan,
Centre d'Etudes Nucléaires de Saclay, DPhPE, 91191 Gif-sur-Yvette, France

A. Le Yaouanc, L. Oliver, O. Pène and J.-C. Raynal
*Laboratoire de Physique Théorique et Hautes Energies**
Université de Paris XI, Bâtiment 211, 91405 Orsay Cedex, France

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O. Pène

ABSTRACT

We point out that current estimates of form factors fail to explain the non-leptonic decays $B \rightarrow \psi K(K^*)$ and that the combination of data on the semi-leptonic decays $D \rightarrow K(K^*)\ell\nu$ and on the non-leptonic decays $B \rightarrow \psi K(K^*)$ (in particular recent polarization data) severely constrain the form (normalization and q^2 dependence) of the heavy-to-light meson form factors, if we assume the factorization hypothesis for the latter. From a simultaneous fit to $B \rightarrow K^{(*)}\psi$ and $D \rightarrow K^{(*)}l\nu$ data we find that strict heavy quark limit scaling laws applied to the form factors do not hold when going from D to B and must have large corrections that make softer the dependence on the masses. This is in contrast with the matrix elements themselves which are found to need smaller $1/m_Q$ corrections to the asymptotic heavy quark scaling laws. We also find that $A_1(q^2)$ should increase slower with q^2 than A_2, V, f_+ . We propose a simple parametrization of these corrections based on a quark model or on an extension of the heavy-to-heavy scaling laws to the heavy-to-light case, complemented with an approximately constant $A_1(q^2)$. This model may be viewed as assuming a precocious validity of strict heavy quark scaling laws for the current matrix elements. Although this model reproduces qualitatively the wanted features for mass and q^2 dependence, and thus reduces the discrepancy with data, it is insufficient to reach a full agreement with the experimental polarization. In our opinion the puzzle is still there.

1. Introduction.

We heard¹ a nice talk about the SLAC B-factory and the BaBar detector which are planned mainly to detect CP violation in B meson decays.

We will not repeat why the search for CP violation is of crucial theoretical interest in a period where only one type of laboratory experiment, namely $K_L \rightarrow \pi\pi$ has

*Laboratoire associé au Centre National de la Recherche Scientifique - URA 63

positively found CP violation[†]. The B system presents this peculiarity that CP violation is commonly believed to lie within experimental reach, in the Standard Model, while non-standard surprises may also happen. This has motivated the building of B-factories.

The so-called angle β of the unitarity triangle should be measurable through the $B \rightarrow \psi K_S$ CP asymmetries and/or through the $B \rightarrow D^{(*)}\overline{D}^{(*)}$ ones². The angle α will not be so easy to measure, and the angle γ is still a challenge. Adding several channels as in² increases the statistics provided that the relative signs of the different channel are such as not to wash out the asymmetry. In the case of $B \rightarrow D^{(*)}\overline{D}^{(*)}$ decays, the Heavy Quark Symmetry (HQS) helps to establish these relative signs which turn out not to dilute the CP asymmetry. In ref.³ we considered applying the same trick to $B_s \rightarrow K^{(*)}D_s^{(*)}$ decay channels. In this case HQS is not so helpful.

We used the factorization assumption and could argue about the signs of the leptonic and semi-leptonic form-factors. However, one crucial expression comes in, of the form $A_1 - cA_2$ where $A_{1,2}$ are semileptonic form factors and c is a kinematical factor whose precise value does not matter here. Using some models, or some arguments from the HQS applied to heavy-to-light decays, we found that the sign of this expression was not easy to settle. At this point we realized that the same expression $A_1 - cA_2$ with a slightly different kinematical factor c also appeared in the expression for the polarization in $B \rightarrow \psi K^*$, a quantity on which several recent and rather precise experimental data exist.

We thus started to look closer at $B \rightarrow \psi K^{(*)}$ decays and we encountered *quite a surprise*. It became obvious to us^{4,5} that the most popular models which are commonly used in heavy flavor decays as well as the simple-minded application of HQS extrapolated from D semi-leptonic decays severely fail to explain $B \rightarrow K^*\psi$ polarization and the ratio $\Gamma(B \rightarrow \psi K^*)/\Gamma(B \rightarrow \psi K)$. This conclusion was independently reached by Gourdin, Keum and Pham⁶.

Such a general failure casts some doubt on the predictions one can extract from the same models and simple-minded ideas whenever some precision is wanted as it was the case in our above-mentioned example of the sign of $A_1 - cA_2$ and the consecutive non dilution of CP asymmetries. As long as the puzzle will not be solved, it will at least demand the severest care and the systematic use of $B \rightarrow \psi K^*$ as a touchstone. We tried to stick to such an attitude in³ and we believe we could safely deliver a positive answer about the non-dilution of the CP asymmetry.

We can also take $B \rightarrow \psi K^{(*)}$ data as a precious source of phenomenological information in the domain of B decays where precise data are still missing. In the following we will first try to convince you that there is really a problem. We discuss next what positive knowledge can be extracted from these polarization data. Of course, this needs some assumption: we will stick to the factorization assumption.

$B \rightarrow \psi K^*$ polarization measurements turn out to be a very efficient touchstone of our present understanding of the non-leptonic decays of B mesons.

[†]Let us leave aside the question of baryogenesis which does not seem to fit easily in the Standard Model.

2. $B \rightarrow \psi K^{(*)}$ data are hardly compatible with current estimates.

To be definite, let us write the form factors :

$$\begin{aligned}
\langle P_f | V_\mu | P_i \rangle &= \left(p_\mu^f + p_\mu^i - \frac{m_i^2 - m_f^2}{q^2} q_\mu \right) f_+(q^2) + \frac{m_i^2 - m_f^2}{q^2} q_\mu f_0(q^2), \\
\langle V_f | A_\mu | P_i \rangle &= (m_f + m_i) A_1(q^2) \left(\varepsilon_\mu^* - \frac{\varepsilon^* \cdot q}{q^2} q_\mu \right) \\
&\quad - A_2(q^2) \frac{\varepsilon^* \cdot q}{m_f + m_i} \left(p_\mu^i + p_\mu^f - \frac{m_i^2 - m_f^2}{q^2} q_\mu \right) + 2m_f A_0(q^2) \frac{\varepsilon^* \cdot q}{q^2} q_\mu, \\
\langle V_f | V_\mu | P_i \rangle &= i \frac{2 V(q^2)}{m_f + m_i} \varepsilon_{\mu\nu\rho\sigma} p_i^\nu p_f^\rho \varepsilon^{*\sigma}.
\end{aligned} \tag{1}$$

where we use the convention $\epsilon^{0123} = 1$.

Using factorization^{7,8,9}, one obtains the following amplitudes in the B meson rest frame:

$$A(\bar{B}_d^0 \rightarrow \psi K) = -\frac{G}{\sqrt{2}} V_{cb} V_{cs}^* 2 f_\psi m_B f_+(m_\psi^2) a_2 p, \tag{2}$$

$$A^{pv}(\bar{B}_d^0 \rightarrow \psi(\lambda=0) K^*(\lambda=0)) = -\frac{G}{\sqrt{2}} V_{cb} V_{cs}^* m_\psi f_\psi,$$

$$\left[(m_B + m_{K^*}) \left(\frac{p^2 + E_{K^*} E_\psi}{m_{K^*} m_\psi} \right) A_1(m_\psi^2) - \frac{m_B^2}{m_B + m_{K^*}} \frac{2p^2}{m_{K^*} m_\psi} A_2(m_\psi^2) \right] a_2, \tag{3}$$

$$A^{pv}(\bar{B}_d^0 \rightarrow \psi(\lambda=\pm 1) K^*(\lambda=\pm 1)) = -\frac{G}{\sqrt{2}} V_{cb} V_{cs}^* m_\psi f_\psi (m_B + m_{K^*}) A_1(m_\psi^2) a_2, \tag{4}$$

$$A^{pc}(\bar{B}_d^0 \rightarrow \psi(\lambda=\pm 1) K^*(\lambda=\pm 1)) = \pm \frac{G}{\sqrt{2}} V_{cb} V_{cs}^* m_\psi f_\psi \frac{m_B}{m_B + m_{K^*}} 2V(m_\psi^2) a_2 p. \tag{5}$$

These amplitude are all proportional to a_2 , i.e. they belong to the so-called class II decays. We see that the non-leptonic data plus the factorization hypothesis can give us information on the form factors at a different kinematic point ($q^2 = m_\psi^2$) than the data on semi-leptonic D decays (small q^2) or the heavy quark limit QCD scaling laws (close to q_{max}^2).

The data for the total rates¹⁰ are:

$$BR(\bar{B}_d^0 \rightarrow \psi K^0) = (7.5 \pm 2.4 \pm 0.8) \times 10^{-4}$$

$$BR(B_d^0 \rightarrow \psi K^{*0}) = (16.9 \pm 3.1 \pm 1.8) \times 10^{-4}$$

$$BR(B^- \rightarrow \psi K^-) = (11.0 \pm 1.5 \pm 0.9) \times 10^{-4}$$

$$BR(B^- \rightarrow \psi K^{*-}) = (17.8 \pm 5.1 \pm 2.3) \times 10^{-4}$$

and the recent results of ARGUS¹¹, CLEO¹¹ and CDF¹² concerning the K^* polarization in the $\bar{B}_d \rightarrow \psi K^{*0}$ decay, are:

$$\begin{aligned} \Gamma_L/\Gamma_{tot} &> 0.78 \text{ (95 \% C.L.)} && \text{ARGUS} \\ \Gamma_L/\Gamma_{tot} &= 0.80 \pm 0.08 \pm 0.05 && \text{CLEO} \\ \Gamma_L/\Gamma_{tot} &= 0.66 \pm 0.10_{-0.08}^{+0.10} && \text{CDF} \end{aligned} \quad (6)$$

where Γ_L is the partial width for the longitudinal polarization whose amplitude is given by (3).

As already pointed out, these decays are affected by the phenomenological factor a_2 which is not well known. To avoid this uncertainty, we will consider the ratio of the total rates

$$R \equiv \frac{\Gamma(\bar{B}_d^0 \rightarrow \psi K^{*0})}{\Gamma(\bar{B}_d^0 \rightarrow \psi K^0)} = 1.64 \pm 0.34 \quad \text{CLEO II}^{13}, \quad (7)$$

and the polarization ratio for ψK^{*0} :

$$R_L \equiv \frac{\Gamma_L(\bar{B}_d^0 \rightarrow \psi K^{*0})}{\Gamma_{tot}(\bar{B}_d^0 \rightarrow \psi K^{*0})}, \quad (8)$$

which are independent of a_2 .

From these formulae one can already conclude qualitatively that :

- i) To get R_L sufficiently large, one needs V/A_1 and A_2/A_1 to be small enough.
- ii) To get R not too large f_+/A_1 must not be too small.

We will consider the predictions for these ratios from the following theoretical schemes :

- 1. Pole model of Bauer, Stech and Wirbel (BSWI)⁸.
- 2. Pole-dipole model of Neubert et al. (BSWII)⁹.
- 3. Quark model of Isgur, Scora, Grinstein and Wise (ISGW)¹⁴.
- 4. QCD sum rules (QCDSR)¹⁵.
- 5. Lattice QCD¹⁷.

The results are given in Table 1. The conclusion is that there is a problem for all known theoretical schemes since both ratios R and R_L cannot be described at the same time. A priori there are three possible explanations :

	$\frac{\Gamma(K^*)}{\Gamma(K)}$	$\frac{\Gamma_L}{\Gamma_{tot}}$	$\frac{A_2^{sb}(m_\psi^2)}{A_1^{sb}(m_\psi^2)}$	$\frac{V^{sb}(m_\psi^2)}{A_1^{sb}(m_\psi^2)}$	$\frac{f_+^{sb}(m_\psi^2)}{A_1^{sb}(m_\psi^2)}$
BSWI ⁸	4.23	0.57	1.01	1.20	1.23
BSWII ⁹	1.61	0.36	1.41	1.77	1.82
ISGW ¹⁴	1.71	0.07	2.00	2.58	2.30
QCDSR ^{15,16}	7.60	0.36	1.19	2.66	1.77
Lattice (a)	3.5 ± 2.5	0.47 ± 0.11			
Lattice (b)	1.9 ± 1.4	0.27 ± 0.16			
Our Ansatz	2.15	0.45	1.08	2.16	1.86
CLEO II ^{11,13}	1.64 ± 0.34	0.8 ± 0.1			
CDF ¹²	-	0.66 ± 0.1			

Table 1: *Comparison of different models, a QCD Sum Rules calculation, a lattice calculation, and our preferred Ansatz (as defined later on) to experiment. The mass and q^2 extrapolation for lattice results are detailed in ref.¹⁷. Lattice (a) uses a kinematical pole in A_2/A_1 while Lattice (b) uses a constant A_2/A_1 as q^2 varies.*

- i) The theoretical schemes for *form factors* are to be blamed for the failure.
- ii) The experimental numbers are not to be trusted too much.
- iii) The basic BSW factorization assumption, which allows to relate $B \rightarrow K^{(*)}\psi$ to the form factors, is wrong for class II decays.

In section 3, we will explore the first possibility on general grounds using the data on $D \rightarrow K^{(*)}l\nu$ and $B \rightarrow K^{(*)}\psi$ combined through HQS. To make the study more quantitative, we will propose a reasonable Ansatz which presents in our opinion the general features that are favored by phenomenology and by some theoretical considerations. This Ansatz, although meant to reconcile both sets of data without violating HQS, still leaves a 2-3 σ discrepancy between $D \rightarrow K^{(*)}l\nu$ and $B \rightarrow K^{(*)}\psi$ data.

Besides the experimental failure apparent in table 1, the quoted popular quark models also present theoretical problems⁵: the BSW models do not satisfy the heavy quark scaling laws when the mass goes to infinity, while the ISGW satisfies only the heavy-to-light scaling law when the initial mass goes to infinity, failing to satisfy the heavy-to-heavy scaling laws when both the initial and final masses go to infinity. We will not elaborate further on this issue in this talk.

3. Heavy Quark Symmetry and the relation to D decays.

3.1. Setting the Problem.

Our aim was to perform a combined experimental and theoretical study of form factors, simultaneously for both of $D \rightarrow K^{(*)}l\nu$ and $B \rightarrow K^{(*)}\psi$ decays, assuming factorization for the latter. We wanted to proceed as independently as possible of the detailed theoretical approaches, using *general Ansätze that respect the heavy-to-light asymptotic scaling laws*, some of them being complemented by ideas derived from

heavy-to-heavy scaling law formulae. Only guided by rigorous theoretical laws and some commonly admitted theoretical prejudices, we will try to display general trends suggested by the experiment. However it turns out that experiment, as it stays today, is not easy to account for in a theoretically reasonable manner. We will also advocate the use of a Quark Model inspired Ansatz, eq. (16), an extension of some heavy-to-heavy scaling relations to the heavy-to-light system. Although not fully successful, this model is able to account roughly for a large set of data.

First, let us review available data. Besides the indirect indications coming from the above $B \rightarrow \psi K(K^*)$ non leptonic data complemented by the BSW factorization assumption, there are data on the $D \rightarrow K(K^*)\ell\nu$ form factors, mainly around $q^2 = 0$. We shall use the world average¹⁸:

$$\begin{aligned} f_+^{sc}(0) &= 0.77 \pm 0.08 \\ V^{sc}(0) &= 1.16 \pm 0.16 \\ A_1^{sc}(0) &= 0.61 \pm 0.05 \\ A_2^{sc}(0) &= 0.45 \pm 0.09 \end{aligned} \tag{9}$$

$$\begin{aligned} V^{sc}(0)/A_1^{sc}(0) &= 1.9 \pm 0.25 \\ A_2^{sc}(0)/A_1^{sc}(0) &= 0.74 \pm 0.15 \end{aligned} \tag{10}$$

As to the q^2 dependence the indications are poor except for the f_+ form factor, where good indications seem to support the relevant vector meson pole dominance. We have used⁵ these indications for the q^2 dependence to advocate a pole-like q^2 dependence of f_+ , A_2 and V , and a flat A_1 . However, in this talk we will concentrate on the evolution from D to B of the *ratios* between different form factors (A_2/A_1 , V/A_1 , etc.) for which we can formulate more general statements. The advantage of discussing first the ratios is that we can draw more direct conclusions from the data before considering absolute branching ratios which also involve the unknown a_2 parameter.

3.2. Asymptotic Scaling Laws for the Heavy-to-light Form Factors.

What can be learned from the theory ? The only exact results take the form of asymptotic theorems¹⁹ valid for the initial quark mass m_Q large with respect to the typical scale of QCD, Λ , to the final meson mass, m_f , and to the final momentum, \vec{q}^\dagger

$$\frac{\hat{f}_+(\vec{q}^2)}{c_+}, \quad \frac{\hat{V}(\vec{q}^2)}{c_V}, \quad \frac{\hat{A}_2(\vec{q}^2)}{c_2} = m_Q^{\frac{1}{2}} \left(1 + O\left(\frac{\Lambda}{m_Q}\right) + O\left(\frac{|\vec{q}|}{m_Q}\right) + O\left(\frac{m_f}{m_Q}\right) \right)$$

[‡]Unless specified otherwise, we use the initial meson rest frame.

$$\frac{\hat{A}_1(\vec{q}^2)}{c_1} = (m_Q)^{-\frac{1}{2}} \left(1 + O\left(\frac{\Lambda}{m_Q}\right) + O\left(\frac{|\vec{q}|}{m_Q}\right) + O\left(\frac{m_f}{m_Q}\right) \right) \quad (11)$$

where the c_+, c_2, c_V, c_1 are unknown constants and where we have used “hats” on form factors to indicate that they depend on the *three-momentum, the natural variable in the heavy-to-light case*:

$$\hat{f}(\vec{q}^2) = f(q^2), \quad \text{with} \quad \vec{q}^2 = \left(\frac{m_i^2 + m_f^2 - q^2}{2m_i} \right)^2 - m_f^2 \quad (12)$$

The asymptotic scaling law (11) allows to relate the form factors, say $D \rightarrow K$ and $B \rightarrow K$, at small recoil $|\vec{q}| \ll m_D$ (i.e. close to q_{max}^2 for each process). In particular, A_2/A_1 scales like m_Q . This has dramatic consequences on the $B \rightarrow \psi K^*$ polarization as we shall now see.

3.3. Failure of the Simple-minded Extrapolation from D to B given the Asymptotic Scaling Laws.

Consider the extrapolation from $D \rightarrow K^{(*)}l\nu$ data at $q^2 = 0$, according to the heavy-to-light asymptotic scaling law. Since the momentum \vec{q} is different in the two above-mentioned sets of data, an hypothesis on the q^2 dependence is needed.

The ratio Γ_L/Γ_{tot} is given by:

$$\frac{\Gamma_L(B \rightarrow K^*\psi)}{\Gamma_{tot}(B \rightarrow K^*\psi)} = \frac{\left(3.162 - 1.306 \frac{A_2^{sb}(m_\psi^2)}{A_1^{sb}(m_\psi^2)} \right)^2}{2 \left[1 + 0.189 \left(\frac{V^{sb}(m_\psi^2)}{A_1^{sb}(m_\psi^2)} \right)^2 \right] + \left(3.162 - 1.306 \frac{A_2^{sb}(m_\psi^2)}{A_1^{sb}(m_\psi^2)} \right)^2} \quad (13)$$

where the indices *sb* indicate that we deal with the $b \rightarrow s$ form factors.

From this expression it is apparent that A_2/A_1 must not be too large[§] in view of the large experimental value of R_L (6), all the more if V/A_1 is large. For example, setting $V = 0$ we get the very conservative upper bound $A_2/A_1 \leq 1.3$ for $R_L > 0.5$. For a more realistic value of $V/A_1 \simeq 2$, the upper bound becomes $A_2/A_1 \leq 1$. Now, according to strict application of the asymptotic scaling laws described above, A_2/A_1 (V/A_1) would be multiplied at fixed \vec{q} by $m_B/m_D = 2.83$. From the central experimental D value, $A_2^{sc}/A_1^{sc} = 0.74$ ($V^{sc}/A_1^{sc} = 1.9$), one gets $A_2^{sb}/A_1^{sb} = 2.09$ ($V^{sb}/A_1^{sb} = 5.38$) at $q^2 = 16.56$ GeV (corresponding in B decay to the same \vec{q}^2 as $q^2 = 0$ in D decay). This is in drastic contradiction with experiment unless there is an unexpectedly strong q^2 variation down to $q^2 = m_\psi^2$. A naive insertion of these values in eq. (13) would indeed give $R_L = 0.014$ which is 4 to 5 sigmas away from the most favorable CDF value. Clearly the message is that *a softening of the increase with respect to the asymptotic scaling law is required*.

[§]Strictly speaking very large values, $A_2/A_1 \geq 3.9$, could also account for a large R_L , but these are unrealistic.

It is now useful to consider the product $R(1 - R_L)$ where the definitions of eqs (7) and (8) have been used:

$$R(1 - R_L) = 2.162 \left(\frac{A_1^{sb}(m_\psi^2)}{f_+^{sb}(m_\psi^2)} \right)^2 \left[1 + 0.189 \left(\frac{V^{sb}(m_\psi^2)}{A_1^{sb}(m_\psi^2)} \right)^2 \right] \quad (14)$$

This gives a lower bound for f_+/A_1 . For the conservative upper bounds $R \leq 2.5$ and $1 - R_L \leq 0.5$, and setting still more conservatively V to zero, we get $f_+/A_1 \geq 1.32$. For a more realistic estimate, $V/A_1 \simeq 2$, and $R \leq 2.0$ we get $f_+/A_1 \geq 1.9$. Contrarily to our discussion in the preceeding paragraph, we find here a lower bound which in itself is compatible with the hard scaling behaviour but not with such a soft scaling as required for A_2/A_1 (remember we had $A_2/A_1 \leq 1.3$ for $V = 0$ and for a more realistic V/A_1 , $A_2/A_1 \leq 1$). *Clearly the trend for f_+/A_1 is somewhat opposite to the one for A_2/A_1 .* To solve this problem, Cheng and Tseng²⁰ have assumed a monopole form for f_+ and A_1 and a dipole form for A_2 and V . This implies a pole behavior for A_2/A_1 leading to a reduced A_2^{sb}/A_1^{sb} and a constant f_+/A_1 , thus keeping it large enough.

3.4. Reminder about Asymptotic Scaling Laws for Heavy-to-heavy Transitions.

It is well known that a much stronger set of relations than the one in subsection 3.2 comes from the Isgur-Wise scaling laws²¹ for transition form factors between two heavy quarks. Using the notations in²² :

$$\begin{aligned} \frac{\sqrt{4m_{P_i}m_{P_f}}}{m_{P_i} + m_{P_f}} f_+(q^2) &= \frac{\sqrt{4m_{P_i}m_{P_f}}}{m_{P_i} + m_{P_f}} \frac{f_0(q^2)}{1 - \frac{q^2}{(m_{P_i} + m_{P_f})^2}} = \frac{\sqrt{4m_{P_i}m_{V_f}}}{m_{P_i} + m_{V_f}} V(q^2) = \\ &= \frac{\sqrt{4m_{P_i}m_{V_f}}}{m_{P_i} + m_{V_f}} A_0(q^2) = \frac{\sqrt{4m_{P_i}m_{V_f}}}{m_{P_i} + m_{V_f}} A_2(q^2) = \frac{\sqrt{4m_{P_i}m_{V_f}}}{m_{P_i} + m_{V_f}} \frac{A_1(q^2)}{1 - \frac{q^2}{(m_{P_i} + m_{V_f})^2}} = \xi(v_i.v_f) \end{aligned} \quad (15)$$

for m_{P_i} , m_{P_f} and m_{V_f} much larger than the typical QCD scale, Λ . In the same limit m_{P_f} and m_{V_f} are equal, and our writing of different masses is only meant for later use in the real subasymptotic regime, where they are very different ($m_K \neq m_{K^*}$).

The denominator that divides $A_1(q^2)$ is a straightforward consequence of the heavy quark symmetry and of the definition of the different form factors. It has not the meaning of a dynamical pole related to some intermediate state. It is still in the mathematical sense a pole of the ratio $A_2(q^2)/A_1(q^2)$ etc, and we shall call it for simplicity the “kinematical pole”.

In the domain of mass with $\Lambda \ll m_f \ll m_{P_i}$ the heavy-to-heavy scaling relations (15) imply the heavy-to-light scaling relations (11) with m_Q substituted by m_{P_i} . Being

more restrictive, eqs. (15) also provide the q^2 dependence of the ratios of form factors and the $O(m_f/m_{P_i})$ corrections⁵.

An essential effect displayed by these $O(m_f/m_{P_i})$ is that they soften the asymptotic scaling relation (11), i.e. they lead to a slower increase (decrease) of A_2 , V , f_+ (A_1) when the initial mass m_{P_i} increases at fixed \vec{q} .

Another welcome consequence of the Isgur-Wise relations (15) is that they fix the ratios of form factors for the same quark masses and the same transfer q^2 , in such a way as to smoothen further the initial mass dependence of the ratios when q^2 decreases. This is clearly illustrated at $q^2 = 0$ where all the form factors are equal and their ratios, equal to 1, do not depend on the masses.

3.5. Our Quark Model Inspired Ansatz.

We now formulate our model based on an extension of the heavy-to-heavy scaling relations (15). Let us first assume that we are in a situation described in the preceding section with two heavy quarks and $m_i \gg m_f \gg \Lambda$. The form factors obey the heavy-to-light scaling relations (11) with specific form factor ratios and specific $O(m_f/m_i)$ corrections. To these, one should also add the unknown $O(\Lambda/m_f)$ corrections to the heavy quark symmetry.

Let us now consider the intermediate region where the final quark ceases to be heavy. Our ignorance comes from the fact that the $O(\Lambda/m_f)$ corrections become large and may totally modify the above mentioned specific relations. Our hypothesis will be that it is not so, i.e. that using *some* of the features of eq. (15) is indeed a good approximation. This hypothesis, although admittedly arbitrary, may be empirically justified to some extent as we shall see. Theoretical arguments in favor of the present Ansatz will come below and in section 3.6.

An unrestricted extension of Isgur Wise formulae (15) cannot describe quantitatively the form factors for a simple reason: the $D \rightarrow K^{(*)} l \nu$ form factors at $q^2 = 0$, eq. (9), are obviously not equal to each other. To account for that we introduce some rescaling parameters (r_+ , r_V , r_1 , r_2), that we assume to be independent of the initial heavy quark mass and of q^2 . We therefore propose the following Ansatz that stays as close as possible of the heavy-to-heavy scaling relations:

$$\begin{aligned} \frac{m_{P_i} + m_{P_f}}{\sqrt{4m_{P_i}m_{P_f}}} \left[1 - \frac{q^2}{(m_{P_i} + m_{P_f})^2} \right] \frac{f_+(q^2)}{r_+} &= \frac{m_{P_i} + m_{V_f}}{\sqrt{4m_{P_i}m_{V_f}}} \left[1 - \frac{q^2}{(m_{P_i} + m_{V_f})^2} \right] \frac{V(q^2)}{r_V} = \\ &= \frac{m_{P_i} + m_{V_f}}{\sqrt{4m_{P_i}m_{V_f}}} \left[1 - \frac{q^2}{(m_{P_i} + m_{V_f})^2} \right] \frac{A_2(q^2)}{r_2} = \frac{m_{P_i} + m_{V_f}}{\sqrt{4m_{P_i}m_{V_f}}} \frac{A_1(q^2)}{r_1} = \eta(\vec{q}, m_f) \quad (16) \end{aligned}$$

where m_f is as usual the final meson mass: m_{P_f} or m_{V_f} . In fact, to conform with the asymptotic Isgur-Wise heavy-to-heavy scaling, the rescaling parameters r_+ , r_V , r_2 , $r_1 = 1 + O(\Lambda/m_f)$, should depend on the final active quark mass m_{q_f} and

tend to one when it goes to infinity, but this is irrelevant here since the final quark will remain the s -quark all over this study.

This Ansatz has the wanted features of yielding softened heavy-to-light scaling relations and a welcome q^2 dependence: $A_1(q^2)$ decreases as compared to the other form factors when q^2 increases.

3.6. Theoretical justifications of our Ansatz

a) Quark model: How do we justify this Ansatz and in particular this “rescaling” procedure? We are mainly motivated by the fact that the general structure of the Isgur-Wise relations (15) also appears in the heavy to light case in a quark model with weak binding treatment, the Orsay Quark Model (OQM)^{23,24,25,26,5}. It gives the kinematical pole factor, differentiating f_+ , A_2 and V from A_1 . It also displays the $O(m_f/m_i)$ corrections predicted by the heavy-to-heavy scaling laws. On the other hand the quark model analysis leads to expect two types of $O(\Lambda/m_f)$ corrections:

i) Corrections taking into account the finite mass of the spectator quark, which are present in the weak binding treatment.

ii) Corrections to the weak binding limit, not included in the OQM.

In this model the dominant correction to asymptotic scaling and the dominant features of q^2 dependence are represented by the Ansatz (16), while additional corrections are present but are small.

b) $B \rightarrow K^*\gamma$: An amusing example that exhibits the same trends as we advocate is provided by the $B \rightarrow K^*\gamma$ form factors. Defining the T_i form factors as follows,

$$\begin{aligned} \langle K^*, k, \epsilon | \bar{s} \sigma^{\mu\nu} q_\nu \frac{1 + \gamma_5}{2} b | B, p \rangle = & -2\epsilon_{\mu\nu\lambda\sigma} \epsilon^{*\nu} p^\lambda k^\sigma T_1(q^2) - i \left[\epsilon_\mu^* (m_B^2 - m_{K^*}^2) - \right. \\ & \left. \epsilon^* \cdot q (p + k)_\mu \right] T_2(q^2) - i \epsilon^* \cdot q \left[q_\mu - \frac{q^2}{m_B^2 - m_{K^*}^2} (p + k)_\mu \right] T_3(q^2) \end{aligned} \quad (17)$$

it is well known that, for $q^2 = 0$, using the identity $\sigma_{\mu\nu} \gamma_5 = \frac{i}{2} \epsilon_{\mu\nu\lambda\sigma} \sigma^{\lambda\sigma}$, one obtains the exact relation:

$$T_1(0) = T_2(0). \quad (18)$$

It has also been shown²⁷ that

$$\begin{aligned} T_1(q^2) &= \sqrt{m_Q} \left(1 + O\left(\frac{\Lambda}{m_Q}\right) + O\left(\frac{|\vec{q}|}{m_Q}\right) \right), \\ T_2(q^2) &= \frac{1}{\sqrt{m_Q}} \left(1 + O\left(\frac{\Lambda}{m_Q}\right) + O\left(\frac{|\vec{q}|}{m_Q}\right) \right). \end{aligned} \quad (19)$$

In the heavy-to-heavy case one may also show that

$$\frac{\sqrt{4m_{P_i}m_{V_f}}}{m_{P_i} + m_{V_f}} T_1(q^2) = \frac{\sqrt{4m_{P_i}m_{V_f}}}{m_{P_i} + m_{V_f}} \frac{T_2(q^2)}{1 - \frac{q^2}{(m_{P_i} + m_{V_f})^2}} = \frac{1}{2} \xi(v \cdot v'), \quad (20)$$

which is of course fully compatible with the relation (18).

The novelty here is that *the relation (18) remains exact when the final quark becomes light*. Since the scaling behaviours of the T_1 and T_2 differ in the vicinity of q_{max}^2 , eq. (19), the equality (18) is *a clear indication that the q^2 behaviour of both form factors differs sensibly*. For example a pole dominance hypothesis for both form factors is totally excluded by these relations. Furthermore, an extension of relation (20) to the heavy-to-light domain, as we have suggested in section 3.5, would directly comply with both relations (18) and (19). Lattice calculations may indicate²⁸ a rather flat q^2 dependence of $T_2(q^2)$ except one lattice group²⁹ who finds, on the contrary, a pole-like $T_2(q^2)$. The flat behaviour leads to a $B \rightarrow K^* \gamma$ branching ratio in agreement with experiment, as expected since the long distance contributions, which are overlooked by the lattice, have been estimated³⁰ to be small.

Finally, let us insist that this is by no means a proof of our Ansatz, it is simply a hint that it points towards the right direction.

c) Matrix elements: Some light can be shed on our Ansatz (16), as far as the mass dependence is concerned, by noting that it amounts to assume that *the matrix elements satisfy an uncorrected asymptotic scaling*. To illustrate this point let us consider a final vector meson V_f with a polarization ϵ^T orthogonal to the initial and final meson momenta.

From eqs. (1) and (16) the matrix elements scale as follows:

$$\begin{aligned} \frac{\langle V_f, \epsilon^T, \vec{q} | A_\mu | P_i \rangle}{\sqrt{4m_B m_{V_f}}} &= r_1 \eta(\vec{q}, m_{V_f}) \epsilon_\mu, \\ \frac{\langle V_f, \epsilon^T, \vec{q} | V_\mu | P_i \rangle}{\sqrt{4m_B m_{V_f}}} &= i r_V \frac{\eta(\vec{q}, m_{V_f})}{1 + v_f^0} (\vec{v}_f \times \vec{\epsilon}^T)_\mu, \end{aligned} \quad (21)$$

where $v_f^\mu = p_f^\mu / m_f$ has been used as well as the relation:

$$1 - \frac{q^2}{(m_{P_i} + m_f)^2} = 2 \frac{m_{P_i}(m_f + E_f)}{(m_{P_i} + m_f)^2}. \quad (22)$$

In this example it is clear that the matrix elements scale exactly like $\sqrt{m_{P_i} m_f}$, which is their asymptotic heavy-to-light scaling behaviour. Our claim in favor of the softened scaling, eq. (16), is equivalent to the statement that the matrix element asymptotic scaling laws are not corrected at non-asymptotic masses. In other words *our softened scaling Ansatz is equivalent to a precocious asymptotic scaling of the matrix elements*.

d) QCD sum rules, lattice calculations: We have argued in⁵ that QCD sum rules qualitatively favor^{31,15,32,33,34} the q^2 dependence of the form factor ratios as depicted in eq. (16), i.e. they generally show an increase of the ratios A_2/A_1 , V/A_1 , f_+/A_1 , with q^2 not very different from the increase due to the kinematical pole $1/(1 - q^2/(m_{P_i} + m_f)^2)$.

Lattice calculations¹⁷ on their side, favor a softened heavy-to-light scaling, as a function of the heavy masses, for the leptonic decay constant F_P and for the form factors except A_2 . However, if this result seems strongly established for the leptonic decay constants, the m_Q dependence of the form factors, and particularly A_2 , are not yet known with enough precision to be conclusive.

3.7. Confronting the form factors to experimental ratios.

In ref.⁵ we have performed a simultaneous χ^2 fit of the $B \rightarrow K^{(*)}\psi$ and $D \rightarrow K^{(*)}l\nu$ data, combining them with the help of different Ansätze. The latter Ansätze are devised to compare the most commonly used “natural” assumptions with our favored eq. (16). For instance they correspond to uncorrected asymptotic scaling of the form factors and/or q^2 independent form factor ratios.[¶]

For every Ansatz we have performed a least χ^2 fit⁵. If we only fit the final polarization, i.e. the ratio R_L defined in eq. (8), in $B \rightarrow K^{(*)}\psi$ with the $D \rightarrow K^{(*)}l\nu$ data, our Ansatz (16) gives the smallest χ^2 . This was expected since (16) reduces A_2^{sb}/A_1^{sb} and therefore improves R_L , see eq. (13). However this improvement is insufficient, since the best χ^2 thus obtained is $\sim 2(\sim 4)$ per degree of freedom when R_L is taken from CDF (CLEO II), i.e. a 2-3 σ discrepancy. When we fit both R_L and R , eq. (7), with $D \rightarrow K^{(*)}l\nu$ data, the Ansatz (16) gives a least χ^2 of $\sim 3(\sim 4)$ per d.o.f, not better than that the alternative Ansatz which incorporates also softened scaling, but q^2 independent form factor ratios. The reason is that all our trial Ansätze assume $A_2/A_1 = f_+/A_1$, and, as argued after eq. (14), the R_L data demand a smaller A_2/A_1 while R data demand a larger f_+/A_1 . The χ^2 fit tries a compromise between these opposite trends. Only by relaxing the constraint $A_2/A_1 = f_+/A_1$ can this be cured, as done in ref²⁰.

To summarize:

- As anticipated, the experimental comparison between $B \rightarrow K^{(*)}\psi$ and $D \rightarrow K^{(*)}l\nu$ favors a soft heavy-to-light scaling.
- The best q^2 dependence cannot be selected from this analysis alone. The separated phenomenological study of the K^* final states, as well as several theoretical considerations, tend to favor the existence of the “kinematical pole”. But the consideration of the $\Gamma(B \rightarrow K^*\psi)/\Gamma(B \rightarrow K\psi)$ ratio tends to wash out this conclusion.

[¶]Universal pole dominance, a very popular assumption, implies approximately constant form factor ratios.

- There remains a difficulty to reconcile experimental results in $B \rightarrow K^{(*)}\psi$ and $D \rightarrow K^{(*)}l\nu$ when taking CDF results for R_L ($\chi^2/dof \simeq 3$), which worsens when using CLEO or ARGUS values for R_L . There seems to be also a particular difficulty to fit simultaneously R and R_L . Only fragile indications of possible ways out of these difficulties are known today⁵.

4. Conclusions.

The first requirement for any model is to fulfill the heavy-to-light scaling relations. This is not the case⁵ for the most popular BSW I and BSW II models, notwithstanding their relatively good empirical successes. ISGW does not fulfill the heavy-to-heavy scaling relations and fails very badly for the polarization.

All the approaches we have considered in this paper encounter difficulties in accounting for the $B \rightarrow K^{(*)}\psi$ data, particularly with the large Γ_L/Γ_{tot} (CLEO and ARGUS data). At present it seems safer to keep open three possibilities to get out of this problem.

- Experiment may not have yet delivered its ultimate word, as the variation between different experiments seem to indicate, and it might evolve towards data easier to account for.
- Although we did not discuss the factorization assumption, it should be kept in mind that it rests on no theoretical ground for color suppressed decay channels, as is the case for $B \rightarrow K^{(*)}\psi$. Carlson and Milana³⁶ find a significant correction to factorization within their perturbative QCD inspired model. Gourdin, Keum and Pham³⁵, propose to test factorization in $B \rightarrow \eta_c K^{(*)}$.
- Finally, models may be wrong. This will now be discussed in more details.

Our analysis has allowed to extricate from data some general trends, namely “softened” scaling, a sensibly different q^2 behaviour of A_1 versus A_2, V, f_+ , and A_1 slowly varying with q^2 . Ansätze that take these indications as a guide, obtain better values for Γ_L/Γ_{tot} , and a more reasonable a_2 (see ref.⁵), although there remains a general tendency to underestimate Γ_L/Γ_{tot} with respect to present data. Let us now comment on these general trends.

Data definitely exclude “hard scaling” i.e. the strict application of asymptotic heavy-to-light scaling formulae in the finite mass domain. We have proposed a “softened” Ansatz which is based on an extension of heavy-to-heavy scaling relations down to the light final meson case, with some rescaling. In fact this is equivalent to assuming a precocious scaling for the axial and vector current *matrix elements*. Consequently, the ratio A_2/A_1 does not increase too fast with the heavy mass.

There are indications from lattice calculations, from Quark Model, and to some degree from phenomenology, that V should undergo an even softer scaling⁵.

Another consequence of the above Ansatz, as well as of the Orsay Quark Model is that $A_2/A_1, V/A_1$ and f_+/A_1 should have a pole like behaviour in q^2 , leading to

an increase with q^2 . This improves the agreement with $B \rightarrow K^{(*)}\psi$ data, and seems to be corroborated by QCD Sum Rules calculations.

$D \rightarrow Kl\nu$ experiments seem to show a pole like behaviour for $f_+(q^2)$. Combined with our preceding Ansatz for the ratios, this implies an approximately constant $A_1(q^2)$. This particular q^2 behaviour is corroborated by the Orsay Quark Model, while QCD Sum Rules give a q^2 dependence of A_1 that never increases very fast, although different detailed shapes are proposed. Lattice calculations, within large errors, might give the same indication.

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